Variational Nonequilibrium Statistical Mechanics
Wintersemester 2018/19
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Blatt 5 - Hausaufgabe

Übung am 30. November 2018

Aufgabe 1: Diffusion in a linear potential

A Brownian particle diffuses freely in $x \in (-\infty, \infty)$ under the influence of a linear potential $U(x) = \alpha x$. The particle is initially ($t = 0$) located at $x = 0$.

a) Write down and solve the Smoluchowski equation for the time evolution of the phase space distribution function $\psi(x, t)$.

b) Sketch $\psi(x, t)$ for different times and interpret the result.

c) Calculate the mean position of the particle $\langle x(t) \rangle$. Use the result to estimate the time required for a colloidal silica sphere (diameter $d = 100$ nm, mass density $\rho_c = 2.3$ g/cm^3) in an aqueous solvent ($\gamma = 10^{-6}$ g/s) to sediment to the bottom of a test tube of height $h = 10$ cm.

Hint: Using a suitable change of variables the Smoluchowski equation reduces to the free diffusion equation under no external field which can be solved in the frequency domain.

Aufgabe 2: Ideal gas in an external field

a) Use the force balance equation to determine the equilibrium density profile of an ideal gas of Brownian particles under the influence of a conservative external force.

b) Does the equilibrium state change if instead of Brownian particles we consider an ideal gas of inertial particles?

c) A two dimensional system of non-interacting Brownian particles is confined in a square box of length H with periodic boundary conditions. The one-body velocity field is $\mathbf{v}(x, y) = v_0 \sin(2\pi y/H)\mathbf{e_x} + v_0 \sin(2\pi x/H)\mathbf{e_y}$ and the density profile is given by $\rho(x, y) = \rho_0(1 + \rho_1[\cos(2\pi x/H) - \cos(2\pi y/H)])$. Here v_0, H, ρ_0 and ρ_1 are positive constants. Verify the system is in steady-state and find the conservative and non-conservative components of the external force that sustains the flow.

Blatt 5 - Präsenzübung

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Aufgabe 3: Mean square displacement

A Brownian particle is at the origin at time $t_0 = 0$, i.e. $\psi(\boldsymbol{r}, t_0) = \delta(\boldsymbol{r})$. Calculate the mean square displacement $\langle (\boldsymbol{r}(t))^2 \rangle$ of the particle.

Aufgabe 4: Adjoint operator

Find the adjoint Smoluchowski operator (assume the phase space distribution vanishes at the boundary).