Variational Nonequilibrium Statistical Mechanics
Wintersemester 2018/19
Lectures Prof. M. Schmidt
Tutorials PD Dr. Daniel de las Heras

UNIVERSITÄT BAYREUTH

Blatt 4 - Hausaufgabe

Übung am 23. November 2018

Aufgabe 1: Kinetic stress

a) Show that in equilibrium the divergence of the kinetic stress tensor reduces to the ideal gas diffusion:

$$\nabla \cdot \boldsymbol{\tau} = -k_B T \nabla \rho \tag{1}$$

b) Prove that

$$\sum_i \int d\boldsymbol{r}^N d\boldsymbol{p}^N \left(\nabla_i \ln \psi(\boldsymbol{r}^N, \boldsymbol{p}^N, t)\right) \psi(\boldsymbol{r}^N, \boldsymbol{p}^N, t) \delta(\boldsymbol{r} - \boldsymbol{r}_i) = \nabla \rho(\boldsymbol{r}, t), \tag{2}$$

with ψ the phase space distribution function.

Aufgabe 2: Clausius-Virial

a) Demonstrate the virial theorem, which states that if positions and momenta remain bounded then

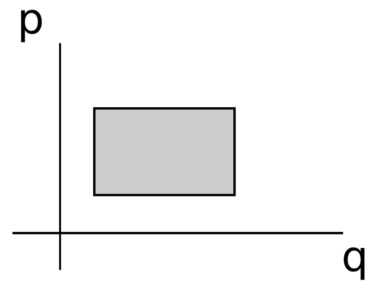
$$\left\langle \sum_i \mathbf{f_i} \cdot \mathbf{r_i} \right\rangle = -2\langle T \rangle. \tag{3}$$

Here T is the total kinetic energy and $\langle \cdot \rangle$ denotes a time average over a time period $\tau \to \infty$.

b) Use the above result to find a relation between the time averages of kinetic and potential energies in a system with a pairwise interparticle potential given by $\phi(r) = \alpha r^n$.

Aufgabe 3: Phase space

a) Find and sketch the trajectories in phase space of a mass m experiencing vertical free fall motion without air resistance. The probability distribution of a collection of identical masses has at a given time a rectangular shape in phase space (see figure). What is the shape of the probability distribution at a later time? Show explicitly that the total area occupied by the probability distribution remains constant over time.



b) A beam of protons has a circular cross section of radius r_1 in the plane perpendicular to the direction of the beam. The momenta are distributed in a circle of radius p_1 in phase space. Using magnets the beam is focused (let $r_2 < r_1$ be the new radius). How does the distribution of momenta change?

Blatt 4 - Präsenzübung

Übung am 23. November 2018

Aufgabe 4: Self adjoint operators

a) Show that $\hat{\boldsymbol{p}}_i, \hat{H}$, $\hat{n}$, $\hat{\mathbf{J}}$, and $\hat{\boldsymbol{\tau}}$ are self-adjoint operators.
b) Let $\hat{\mathbf{B}}$ be a non-self-adjoint operator $\hat{\mathbf{B}} \neq \hat{\mathbf{B}}^\dagger$. Show that

$$\langle \hat{\mathbf{A}} \rangle = \mathrm{Re}\, \langle \hat{\mathbf{B}} \rangle, \tag{1}$$

with $\hat{\mathbf{A}} = \frac{1}{2}(\hat{\mathbf{B}} + \hat{\mathbf{B}}^\dagger)$.