Variational Nonequilibrium Statistical Mechanics
Wintersemester 2018/19
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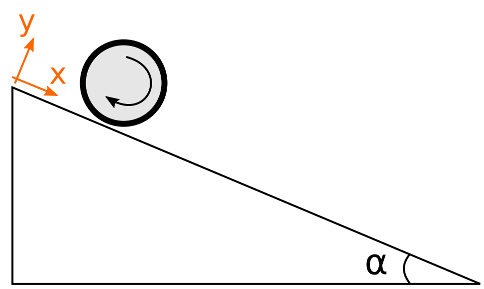
UNIVERSITÄT BAYREUTH

Blatt 2 - Hausaufgabe

Übung am 9. November 2018

Aufgabe 1: Rolling disk

A solid disk of mass M and radius R (moment of inertia $I = 0.5MR^2$) initially at rest rolls without slipping down an inclined plane of inclination α. Find (and solve) the equation of motion of the disk using Hamilton's action principle. That is, formulate the Lagrangian and find the trajectories that make the action stationary.



Aufgabe 2: Hamilton's action principle and Lagrangian mechanics

a) Use Hamilton's principle of stationary action to prove that if two Lagrangians differ only in the total time derivative of a function $f(\mathbf{q}, t)$, then the corresponding equations of motion are the same.

b) Show that if a Lagrangian is invariant under time translation, i.e. $L(\mathbf{q}, \dot{\mathbf{q}}, t) = L(\mathbf{q}, \dot{\mathbf{q}}, t + t_0)$, then it follows that that the Hamiltonian is a constant of motion.

Aufgabe 3: Functional derivatives

Let $\phi(r)$ and $\mathbf{f}(r)$ be a scalar and a vector field, respectively. Calculate the following functional derivatives (integrals run over the entire volume).

$$\frac{\delta}{\delta\phi(\mathbf{r}')} \int d\mathbf{r}\phi(\mathbf{r})[\ln\phi(\mathbf{r}) - 1], \tag{1}$$

$$\frac{\delta}{\delta\phi(\mathbf{r}'')} \int d\mathbf{r} \int d\mathbf{r}'\phi(\mathbf{r})\omega(|\mathbf{r} - \mathbf{r}'|)\phi(\mathbf{r}'), \tag{2}$$

$$\frac{\delta}{\delta\mathbf{f}(\mathbf{r}')} \int d\mathbf{r}\phi(\mathbf{r})f^2(\mathbf{r}), \tag{3}$$

$$\frac{\delta}{\delta\mathbf{f}(\mathbf{r}')} \int d\mathbf{r}\phi(\mathbf{r})\nabla \cdot \mathbf{f}(\mathbf{r}), \tag{4}$$

$$\frac{\delta}{\delta\mathbf{f}(\mathbf{r}')} \int d\mathbf{r}\phi(\mathbf{r})(\nabla \times \mathbf{f}(\mathbf{r}))^2. \tag{5}$$

Blatt 2 - Präsenzübung

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Aufgabe 4: Rolling disk - Constraint forces

Find the constraint forces in exercise 1 (rolling disk) using a Lagrange multiplier for the constraint that links the position and the rotation of the disk.