Variational Nonequilibrium Statistical Mechanics

Wintersemester 2018/19
Lectures Prof. M. Schmidt
Tutorials PD Dr. Daniel de las Heras

UNIVERSITÄT BAYREUTH

Blatt 11 - Hausaufgabe

Übung am 25. Januar 2019

Aufgabe 1: Shear migration

A two-dimensional system of hard disks of diameter σ is subject to shear via a periodic force field $\mathbf{f}_{\text{ext}} = f_0 \sin(2\pi y/L)\mathbf{e}_x$ of periodicity $L \gg \sigma$. Due to the induced migration force (particles migrate to the regions of low shear rate) the density profile in steady state is $\rho(y) = \rho_0(1 + \Delta \sin^2(2\pi y/L))$. Under weak driving conditions ($f_0\sigma/k_BT \ll 1$), the modulation of the density profile is very small, i.e., $\Delta \ll 1$.

b) Write down the steady state force balance in both directions. Find the (approximated) value of the $y-$component of the superadiabatic force in steady state. Hint: Use a simple second virial DFT functional for the excess part.

a) Sketch all the forces in the system and the density profile. Interpret the results.

Aufgabe 2: Adiabatic potential

Under the influence of an external field, a given system reaches a steady state with a density profile $\rho(x) = \rho_0 \exp(-x^2/\sigma^2)$. In the adiabatic system, the corresponding adiabatic potential is $V_{\text{ad}}(x) = \alpha x^2$, with α a positive constant.

a) Find the adiabatic excess force field. Sketch the forces and the density profile in the adiabatic system.

b) What is the interparticle potential if $\alpha = k_BT/\sigma^2$?

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Aufgabe 3:

Show that

$$\frac{\delta \mathcal{G}_t}{\delta q \dot{\mathbf{A}}(\mathbf{r},t)} = \dot{\mathbf{J}}(\mathbf{r},t). \tag{1}$$